

On the detectability of quantum spacetime foam with gravitational-wave interferometers

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Abstract

We discuss a recent provocative suggestion by Amelino-Camelia and others that classical spacetime may break down into “quantum foam” on distance scales many orders of magnitude larger than the Planck length, leading to effects which could be detected using large gravitational wave interferometers. This suggestion is based on a quantum uncertainty limit obtained by Wigner using a quantum clock in a gedanken timing experiment. Wigner’s limit, however, is based on two unrealistic and unnecessary assumptions: that the clock is free to move, and that it does not interact with the environment. Removing either of these assumptions makes the uncertainty limit invalid, and removes the basis for Amelino-Camelia’s suggestion.

Key words: Quantum spacetime foam; gravitational wave detectors; clocks

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1 Introduction

Amelino-Camelia [1,2] has made the interesting suggestion that the fundamental minimum distance uncertainty between two spatially separated points may depend on the separation distance and be many orders of magnitude larger than the Planck length. That is, the classical picture of spacetime may

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break down to “quantum foam” on a surprisingly large scale. This would imply that quantum gravity effects could potentially be probed with current or near-future interferometers designed for use as gravitational-wave detectors.

We consider here the arguments put forward by Amelino-Camelia [1,2], as well as related arguments by Ng and van Dam [3,4]. These all rely on a lower limit on distance measurement uncertainty obtained long ago by Wigner [5], who used a quantum clock in a gedanken light travel timing experiment. The uncertainty limit is the result of spreading of the wave packet of the clock. There are two basic assumptions in the analysis: the quantum clock is free, and it does not interact with its environment during the course of the (macroscopic) timing experiment. That is, the clock evolves according to the unitary operator $U(t, 0) = \exp(-iHt)$ with a strictly free Hamiltonian; there is no interaction with a potential, the environment, or anything else.

We show first that if the clock is quantum mechanical but not free (if it is bound in a harmonic oscillator potential, for example) then the uncertainty limit becomes much smaller than that obtained by Wigner and used by Amelino-Camelia and Ng and van Dam. We then point out that, if the clock is sufficiently large or complex, it will interact with its environment in such a way that its wave function decoheres; that is, loses the phase coherence necessary for superposition into a packet. In addition, such interactions (eg, a restraint system) may localize or “collapse” the wave function. These effects generally happen in a time much less than that needed for macroscopic distance measurements. The result is that the clock wave function does *not spread linearly* over macroscopic times.

It thus appears that the clock used in the gedanken experiments is particularly ill-suited to its purpose. It does not appear to be “the best that we can imagine” [5]. Indeed, due to decoherence, we expect that an ideal quantum clock with nontrivial internal construction is not obtainable, *even in principle*.

Finally, we note that an additional gravitational length uncertainty limit given by Ng and van Dam [3,4] would violate everyday experience (by many orders of magnitude) if taken to be in any way *intrinsic* to the measurement process.

2 Review of the quantum uncertainty limit

The work of Amelino-Camelia [1,2] and of Ng and van Dam [3,4] relies on an analysis of distance measurement following Wigner [5]. The various authors define the spatial distance between two points as $\ell = ct/2$, where t is the time it takes light to complete a round trip between them, and c is of course the velocity of light. At one of the points they place a system of a clock plus a

transmitter-receiver that is used to send a light signal to a mirror at the other point, and to time its return. To obtain a quantum limit on the uncertainty in ℓ they assume the clock system behaves as a free quantum object, and calculate the uncertainty in its position during the transit time of the light. This is a standard problem discussed in quantum mechanics texts, often referred to as spreading of the (minimum uncertainty) wave packet (e.g. [6], p. 64). It may be solved using a superposition of plane wave solutions of the Schrödinger equation, or a simple approximation may be obtained using the uncertainty principle [3,4].

Briefly the uncertainty principle derivation is as follows. (We delete factors of order unity throughout as irrelevant to the discussion.) Denote the initial uncertainty in the clock system position by $\delta\ell_0$ and its mass by m_c . By the uncertainty principle this implies an uncertainty in its velocity of $\delta v \geq \hbar/m_c\delta\ell_0$. Thus the position uncertainty or wave packet width spreads with time, and at t it is $\delta\ell(t) \geq \hbar t/m_c\delta\ell_0$. The two uncertainties $\delta\ell_0$ and $\delta\ell(t)$ combine as independent random variables, to give a net result

$$\delta\ell_Q^2 \geq \delta\ell_0^2 + (\hbar t/m_c\delta\ell_0)^2 \quad (1)$$

(Amelino-Camelia and Ng and van Dam add the two uncertainties linearly, which gives essentially the same final conclusion.) The net uncertainty is minimum at $\delta\ell_0^2 \approx \hbar t/m_c$, so that

$$\delta\ell_Q^2 \geq \frac{\hbar t}{m_c} \approx \frac{\hbar \ell}{m_c c} \quad (2)$$

This quantum uncertainty limit is taken to be *intrinsic* to the length measurement process.

We emphasize, as stated in the introduction, that the uncertainty limit (2) is quite correct for a freely moving quantum clock whose wave function evolves according to the Schrödinger equation during the macroscopic transit time t of the light. We will critically discuss this limit and its relevance in §4.

3 Review of the total uncertainty limit

Amelino-Camelia [1,2] observes that whatever the nature of the clock system, it must have a Schwarzschild radius less than its characteristic size d , or the measurement light could not escape the gravitational field of the clock system and be sent to the mirror; that is, $Gm_c/c^2 \leq d$. Combined with eq. (2) this

gives a total uncertainty limit of

$$\delta\ell_T^2 \geq \frac{\ell}{d} \frac{G\hbar}{c^3} \approx \frac{\ell\ell_P^2}{d}, \quad \delta\ell_T \geq \sqrt{\frac{\ell\ell_P^2}{d}} \quad (\text{Amelino-Camelia}) \quad (3)$$

This equation is the basis of ref. [1], where d is taken to be of order ℓ_P . The suggestion is then made that the uncertainty may be detectable using large interferometers designed for the detection of gravitational waves. Indeed, the uncertainty bound appears already to be violated experimentally.

Ng and van Dam [3,4] are more specific about the clock and assume that it contains two mirrors separated by a distance d , and that a pulse of light bounces back and forth between the mirrors. One tick of the clock takes a time d/c , which they take to be a *fundamental discretization error*, $\delta t \geq d/c$. (That is, they assume that the light pulse cannot be detected while between the mirrors.) This implies an error in the length measurement of $\delta\ell \geq d$. Finally, the assumption is made that the clock system is spherically symmetric and larger than its Schwarzschild radius. Then there is a fundamental error in the length measurement due to the mass of the clock of roughly

$$\delta\ell_G \geq \frac{Gm_c}{c^2} \quad (4)$$

This gravitational uncertainty limit is taken to be *intrinsic* to the length measurement process. Eq. (4) tells us that the fundamental uncertainty in a distance measurement is approximately equal to the local deviation of the spacetime metric from Lorentzian. We will critically discuss this in §5.

Ng and van Dam then combine the quantum and gravitational uncertainty limit and eliminate the clock system mass by multiplying eqs. (2) and (4), which gives

$$\delta\ell_T^3 \geq \ell \frac{\hbar G}{c^3} = \ell\ell_P^2, \quad \delta\ell_T \geq (\ell\ell_P^2)^{1/3} \quad (\text{Ng and van Dam}) \quad (5)$$

This combined uncertainty limit subsumes the individual quantum and gravitational uncertainty limit (2) and (4) in their further discussion.

4 Comments on the quantum uncertainty limit

Eqs. (3) and (5) indicate that spacetime displays quantum foam properties on scales far above the Planck length of 10^{-35} m. For example, a 1 m distance

would be fuzzy to about 10^{-18} m — about the distance presently probed by high energy physics experiments — according to eq. (3), and to about 10^{-23} m according to eq. (5). Could spacetime really be fuzzy on scales this far beyond the Planck length? The key factor is clearly the ℓ in the quantum relation (2). As we noted, this equation is correct provided the clock system is free and evolves according to Schrödinger’s equation during the position measurement.

We first comment on the assumption that the clock system is *free*. This is important in that it leads to the factor of t in the uncertainty limit (2). If we assume the contrary, that the clock is bound to other objects in its vicinity, the uncertainty limit does not follow. As an example, consider a clock bound in a simple harmonic oscillator potential $V = m_c \omega^2 x^2 / 2$. The width of the ground state wave function for such a clock is of order $\delta \ell_Q^2 \approx \hbar / m_c \omega$, and the wave function does *not* spread with time (e.g. [6], p. 73).

In the spirit of the work of the previous authors we may also obtain this result heuristically using the uncertainty principle. We think of the clock as “trying” to settle into a state with zero momentum at the bottom of the potential well, but prevented from doing so by the uncertainty principle, which requires *at least* that $p \geq \hbar/x$. Then the total energy is

$$E = \frac{p^2}{2m_c} + \frac{m_c \omega^2 x^2}{2} \quad (6)$$

which has a minimum at $x^2 = \hbar / m_c \omega$. This represents the position uncertainty of the clock rather than eq. (2). In terms of the period t_o of the oscillator we may thus write the position uncertainty limit as

$$\delta \ell_Q^2 \geq \frac{\hbar}{m_c \omega} = \frac{\hbar t_o}{2\pi m_c} \quad (7)$$

This has the same form as the uncertainty limit obtained by Wigner, but with the time of observation replaced by the period of the oscillator. Whereas the time of observation must be macroscopic (as large as desired!) the period of the oscillator can be made as small as desired in principle — and quite small in reality. For example if the clock is taken to be bound like an atom in a crystal the period would be of order 10^{-12} s. It thus appears that the assumption that the gedanken clock be free is not necessary and leads to an unrealistically large distance uncertainty.

We next comment on the assumption that the clock is truly quantum mechanical, in the sense that it undergoes unitary evolution according to the Schrödinger equation during the macroscopic time of observation. This implies that it be sufficiently isolated from its environment that no significant interactions occur. If it is not so isolated, its wave function will suffer decoher-

ence, which means that the superposition of plane waves loses phase coherence and ceases to form a packet. The decoherence may be caused by interaction with ambient light, air molecule collisions, restraint by a tie-down system, or even interaction of the clock with its *own components*! For almost any system of larger than atomic size, decoherence occurs in a time much shorter than that required for macroscopic distance measurement [7]. In addition, the interaction of the clock with its environment (eg, a tie-down system) may be such that it remains localized. Loosely speaking, in the language of the Copenhagen school, we may think of some interactions as providing position “measurements” on a time scale that is less than macroscopic. A clock that suffers wave function decoherence or is subject to essentially continuous position measurements would not have a linearly increasing position uncertainty, and would thus violate the uncertainty limit (2).

In summary it appears that eq. (2) is only relevant if one chooses to consider a freely moving clock undergoing unitary evolution according to Schrödinger’s equation, with no significant environmental interactions. Such a clock, if composed of internal mirrors or other parts, is probably unobtainable, and even if it could be obtained, would be a very poor clock.

5 Further comments on the gravitational uncertainty limit

Eq. (4) of Ng and van Dam, based on gravity, is interesting but also presents difficulties. The presence of the measurement clock system certainly produces a *distortion* of spacetime, but eq. (4) tells us that it also produces an *uncertainty* in spacetime distances of about the same amount! This is a remarkable statement. Suppose we take it seriously as an intrinsic property of spacetime and not just an artifact of the model clock. Since the spinning Earth is certainly an excellent clock (the oldest and most important one we have) we would conclude that objects in the vicinity of the Earth have a minimum intrinsic position uncertainty of roughly the Schwarzschild radius of the Earth, which is about 1 cm. This is manifestly false by many orders of magnitude.

In general the distances near a massive object of a given configuration may be determined theoretically (by general relativity) and measured (by diverse means), and the two agree with each other to impressive accuracy, far better than eq. (4) would suggest. This is what we mean when we say that general relativity is well tested [8].

6 Comment on the effective hypothetical clock mass

Amelino-Camelia [1,2] and Ng and van Dam [3,4] use the uncertainty limits (3) and (5) to discuss the noise in an interferometer such as the LIGO test model. Both are based on the quantum uncertainty limit (2). Despite the preceding comments let us suppose that eq. (2) is correct. Then for small hypothetical clock masses it is the operative bound, and we may use noise measurements to place a lower limit on the effective mass of the hypothetical clock mass. Following ref. [1] we use eq. (2) and write the variance of the noise as

$$\delta\ell_Q^2 = \frac{\hbar t}{2m_c} = \int_{1/t}^{f_{max}} [S(f)]^2 df \quad (8)$$

where S is the spectral density of the noise due to space time foam in eq. (2). From this it follows that, if the maximum frequency cutoff f_{max} is reasonably large,

$$S(f) = \sqrt{\frac{\hbar}{2m_c}} \frac{1}{f} \quad (9)$$

For the LIGO test model the measured noise limit is about 3×10^{-19} m/Hz^{1/2} at 450 Hz [9], which places a lower limit on the hypothetical clock mass of about

$$m_c \geq \frac{\hbar}{2f^2 S^2} \approx 3 \text{ g} \quad (10)$$

This is a remarkably large mass; it exceeds the mass of the essential working parts of wristwatches that many of us are wearing at this moment. As such, it hardly seems plausible as a fundamental property of spacetime.

7 Summary

Our analysis indicates that the quantum uncertainty limit (2) is based on assumptions that are neither realistic nor necessary. A quantum clock bound in a potential well does much better than the postulated limit, as does a macroscopic clock which interacts continuously with its environment.

In addition, the gravitational uncertainty limit (4) suggested by Ng and van Dam appears to imply a fundamental distance uncertainty of about 1 cm near the surface of the Earth, which is contrary to observation.

Our conclusion is that the uncertainty limits used are artifacts of the choice of a particular type of hypothetical clock, and are non-fundamental in nature. There is thus no reason at present to believe that quantum uncertainty manifests itself at scales very much larger than the Planck length. (For a derivation of the Planck length as the minimum length see refs. [10] and [11].)

We note in closing that we know of only one way that quantum spacetime foam might be detectable in the lab in the relatively near future. Arkani-Hamed, Dimopoulos and Dvali [12] have noted that gravity has been probed in the laboratory only down to distances of about 1 cm. Based partly on considerations of dimensionality, they suggest that gravity may operate quite differently at smaller distances, and that the “effective Planck scale” may consequently be only a little beyond the electroweak energy scale now probed in high energy experiments. Experimental tests of this idea using table-top sized apparatus will soon be under way.

References

- [1] G. Amelino-Camelia, *Nature* 398 (1999) 216.
- [2] G. Amelino-Camelia, *Mod. Phys. Lett. A* 9 (1994) 3415.
- [3] J. Y. Ng, H. van Dam, *Mod. Phys. Lett. A* 9 (1994) 335.
- [4] J. Y. Ng, H. van Dam, *Found. Phys.* (1999), in press; gr-qc/9906003.
- [5] E. P. Wigner, *Rev. Mod. Phys.* 29 (1957) 255.
- [6] L. I. Schiff, *Quantum Mechanics*, 3rd. ed. (McGraw-Hill, 1968).
- [7] There is now a vast literature on the decoherence effect, and the closely related problem of what constitutes a measurement. Of particular physical interest are E. Joos, H. D. Zeh, *Z. Phys.* B59 (1985) 223 and W. H. Zurek, *Phys. Rev.* D24 (1981) 1516. More recent references are R. Omnes, *Phys. Rev.* A56 (1997) 3383 and M. Tegmark, quant-physics/9907009 (1999). Finally, extensive discussion and up-to-date references may be found in R. Omnes, *The Interpretation of Quantum Mechanics* (Princeton University Press, 1994) and D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, H. D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer-Verlag, Heidelberg, 1996).
- [8] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, 1993).
- [9] A. Abramovici et al., *Phys. Lett.* A218 (1996) 157.
- [10] R. J. Adler, D. I. Santiago, *Mod. Phys. Lett.* (1999), in press; gr-qc/9904026.
- [11] C. W. Misner, K. Thorne, J. A. Wheeler *Gravitation* (Freeman, New York, 1973), §43.4.

[12] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B429 (1998) 263.